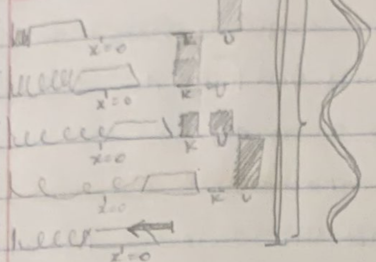
Chapter 15 Textbook Notes - Periodic Motion

Periodic Motion = Any motion that repeats itself at regular time intervals

Vibration or oscillation = back & forth periodic motion

15.1: Periodic Motion & Energy



The cart does not stop at $x=0$ because its potential energy is 0 but its kinetic energy is not 0. When KE = 0, direction reverses. This will happen indefinetely unless energy is dissipated. Dying out of periodic motion due to dissipation of energy = damping.

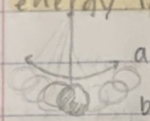
Period (T) = Time interval it takes to complete a full cycle of the motion

Frequency (f) = Inverse of period $\frac{1}{T}$ = number of cycles per second

Amplitude (A) = magnitude of carts maximum displacement from equilibrium position

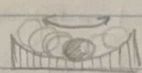
↳ Related to mechanical energy of system $(U+K)$ ⇒ greater initial compression of spring = greater amplitude = greater potential energy

Periodic Motion is characterized by a continuous conversion between potential and kinetic energy in a closed system. Examples:

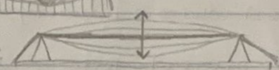


a. pendulum

b. clamped ruler



c. ball in a bowl:



d. string instrument:

a. restoring force = tangential component of gravitational force; assoc w/ gravitational PE

b. restoring force = vertical component of elastic force in ruler; assoc w/ elastic PE

c. restoring force = tangential component of gravitational force; assoc. w/ gravitational PE

d. restoring force = vertical component of elastic force in string; assoc. w/ elastic PE

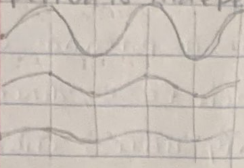
RQ 01:

An object on the end of a spring undergoes simple harmonic motion. At the instant when the object is at its maximum displacement from equilibrium, what is the magnitude of the instantaneous acceleration of the object?

At maximum displacement, the magnitude of the instantaneous acceleration of the object is at its maximum value, since $F=-kx$ (from Hookes Law), so the greater the displacement (x) the greater the magnitude of the force and therefore by $a=\frac{F}{m}$, the greater the magnitude of the acceleration.

15.2 - Simple Harmonic Motion

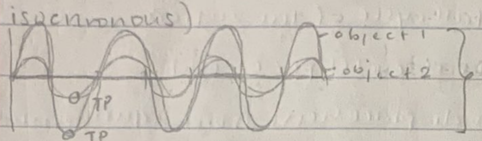
When the amplitude of the motion is not too great ($\sin\theta = \theta$ by small angle approximation) period is independent of amplitude. A system that exhibits equal periods for all amplitudes



= isochronous. This is why the pitch of a guitar note is not dependent on how far back you pull the string when you pluck it.

The $x(t)$ curves for an isochronous are sinusoidal.

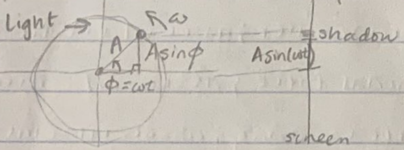
Any periodic motion that yields a sinusoidal $x(t)$ curve is called simple harmonic motion, and a system that executes such motion is called a simple harmonic oscillator (all SHM are isochronous)



Object 1 has twice the amplitude of object 2, travels twice the distance in 1 period that object 2 covers, its velocity is always twice as great as object 2 and therefore its acceleration at each instant must be twice as great as 2's acceleration.

Acceleration is always greatest at turning points. An object executing simple harmonic motion is subject to a linear restoring force that tends to return the object to its equilibrium position and is linearly proportional to the object's displacement from its equilibrium position.

If we project the shadow of a ball moving in a circle at constant speed onto a screen perpendicular to the plane of the motion, the shadow moves up & down in SHM.



The potential energy of a spring is proportional to the square

RQ 02:

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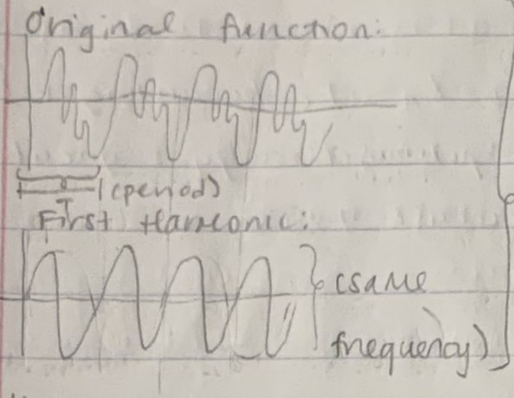
15.3 - Fouriers Theorem

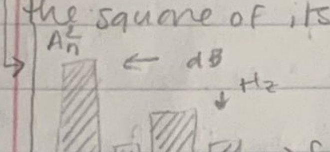
Fouriers Theorem - Any periodic function can be written as a sum of sinusoidal functions of frequency $f_n = \frac{n}{T}$, where $n \geq 1$ is an integer and T is the period of the function. So, any periodic motion can be treated as a superposition of SHMs.

The fundamental frequency or first harmonic is the sinusoidal function component with the same frequency as the original periodic function



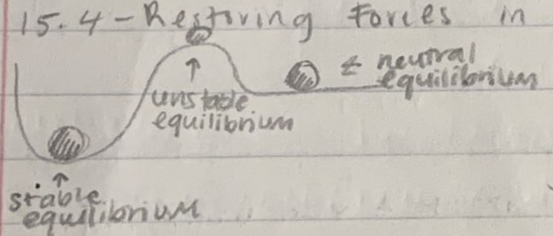
Adding up all the harmonics will create the original function. Breaking a function down into the sinusoidal harmonics (each with a shorter & shorter period (aka higher frequency)) is called Fourier Analysis. The sum of the harmonics is called a Fourier series or Fourier synthesis.

The energy of a simple harmonic oscillator is proportional to the square of its amplitude. So, it is customary to plot A^2 as a function of frequency,



And this type of plot is called the spectrum of the periodic function.

15.4 - Restoring Forces in SHM



In the absence of friction, a small displacement of a system from a position of stable equilibrium causes the system to oscillate

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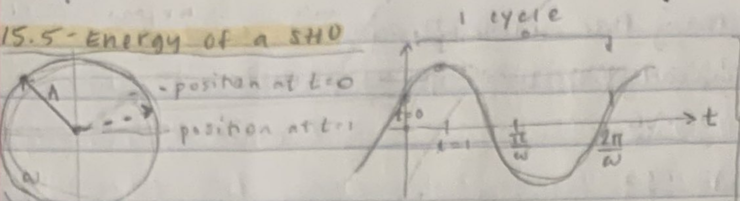
15.5 - Energy of a SHO



A sinusoidally varying, time dependent function describing simple harmonic motion can be represented by a rotating arrow of length A whose tip traces out a circle.

ω = rotational speed of phasor = $\frac{2\pi}{T}$ $\Rightarrow$ $\omega = 2\pi f$ = angular frequency

SI unit of frequency is Hz

SI unit of angular frequency (ω) is s^{-1}

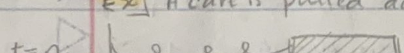
Phase of motion = rotational position of the tip of the phasor relative to the horizontal axis

Phase is represented by the symbol $\phi(t)$. For constant angular frequency, phase increases linearly with time. $\Rightarrow$ $\phi(t) = \omega t + \phi_i$ (where ϕ_i is the initial phase at $t=0$)

Vertical component of A from above function can be written as: $x(t) = A\sin(\omega t + \phi_i)$

where amplitude A is equal to the length of the phasor

Ex) A cart is pulled away from equilibrium position & released at $t=0$. a) what is the initial phase of the cart's oscillation? b) half a period later and



c) one period later? d) what are the carts initial phases when its at its equilibrium position but at $t=0$ its given a push in the negative x-direction to oscillate; and e) The cart is first pulled back a distance d in the +x direction, and then at $t=0$ is given a sharp blow in the +x direction, so that it carries out an oscillation of 2d

a) Since the cart starts at max amplitude in the +x direction, this means the phase is pointing straight up. So $\phi_i = \frac{\pi}{2}$ b) 1/2 a period later it is at a maximum amplitude in the -x direction, so $\phi_i = \frac{3\pi}{2}$. This can also be found using the fact that 1/2 period = 1/2 cycle = $180° \Rightarrow 90°(\phi_i) + 180° = \frac{3\pi}{2}$ c) In one full period, 1 cycle is completed $\Rightarrow \phi_i + 2\pi = \frac{\pi}{2} + 2\pi = \frac{5\pi}{2}$ d) when the cart is in equilibrium, the phasor is horizontal, so its either 0 or π. Since it moves in the -x direction, it must be π, since the phasor always travels counterclockwise e) $x(t) = A\sin(\omega t + \phi_i)$, so at $t=0$, $x(0) = A\sin\phi_i = d$. Since $A = 2d \Rightarrow \sin\phi_i = \frac{1}{2} \Rightarrow \phi_i = \frac{\pi}{6}$ or $\frac{5\pi}{6}$. Since the cart moves in the +x direction at $t=0$, $\phi_i = \frac{\pi}{6}$.

By differentiating x(t) equation, we get v_x & a_x of harmonic oscillator:

$v_x = \frac{dx}{dt} = \omega A\cos(\omega t + \phi_i)$ $\qquad a_x = \frac{d^2x}{dt^2} = -\omega^2 A\sin(\omega t + \phi_i) = -\omega^2 x$

$ma_x = -m\omega^2 x \Longrightarrow \Sigma F_x = -m\omega^2 x$

Work done by forces exerted on the oscillating obj. when it is moving from equilibrium:

$W = \int_{x_0}^{x} \Sigma F_x(x)\,dx = -\int_{x_0}^{x} m\omega^2 x\,dx$ $\Rightarrow$ This work causes kinetic energy to change:

$\Delta K = -m\omega^2 \int_{x_0}^{x} x\,dx = -m\omega^2\left[\frac{1}{2}x^2\right]_{x_0}^{x} = \frac{1}{2}m\omega^2 x_0^2 - \frac{1}{2}m\omega^2 x^2$

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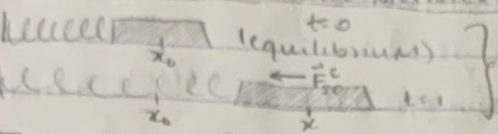
The change in kinetic energy must be caused by some form of potential energy, and since $\Delta U = -\Delta K$: $\Delta U = U(x) - U(x_0) = \frac{1}{2} m\omega^2 x^2 - \frac{1}{2} m\omega^2 x_0^2$

E (total mech. energy) $= K + U = \frac{1}{2} m\omega^2 A^2$

15.6 - Simple Harmonic Motion & Springs

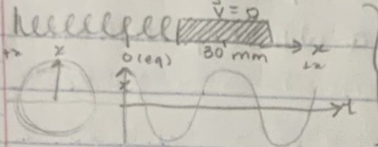


There is an elastic restoring force being exerted on the cart of: $\vec{F}^c_{scx} = -k(x - x_0)$

The angular frequency of this oscillation will be: $\omega = +\sqrt{\frac{k}{M}}$

Motion of the cart is given by: $x(t) = A\sin(\sqrt{\frac{k}{m}}\,t + \phi_i)$

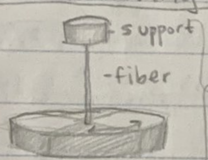
Ex) A cart of mass $m = 0.5$ kg is fastened to a spring of spring constant $k = 14$ N/m is pulled 30 mm away from its equilibrium and then released with 0 initial velocity. What are the carts position & the x-component of velocity 2.0 seconds after being released?



0 initial velocity $\Rightarrow K = 0$ at $x = 30$ mm $\Rightarrow$ initial displacement = max displacement $\Rightarrow A = 30$ mm $\Rightarrow$ initial phase is $\phi_i = +\frac{\pi}{2}$

To determine position of cart at $t = 2.0$ s, use: $x(t) = A\sin(\omega t + \phi_i)$. To determine angular frequency ω, use $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{14\text{ N/m}}{0.5\text{ kg}}} = \sqrt{28\text{ s}^{-1}} = 5.3\text{ s}^{-1}$. Substitute into $x(t)$ equation of: $x(2.0\text{ s}) = 30\text{ mm}\sin[(5.3\text{ s}^{-1})(2.0\text{ s}) + \frac{\pi}{2}] = \boxed{-12\text{ mm}}$. Then plug into v_x equation: $v_x(2.0\text{ s}) = (5.3\text{ s}^{-1})(30\text{ mm})\cos[(5.3\text{ s}^{-1})(2.0\text{ s}) + \frac{\pi}{2}] = \boxed{+1.5 \times 10^2\text{ mm/s}}$.

15.7 - Restoring Torques



A torsional oscillator is an example of rotational rather than translational displacement. If the disk is twisted, elastic potential energy is stored in the fiber. When the disk is released it oscillates its energy back & forth between elastic PE & rotational KE

$\Sigma \tau_\theta = I\alpha_\theta$ (sum of torques on disk = rotational inertia of disk x its rotational acceleration)

If a torsional oscillator of rotational inertia I is twisted through a small angle from its equilibrium position θ_0, the restoring torque is: $\tau_\theta = -\kappa(\theta - \theta_0)$ (where κ is torsional constant).

When $\theta_0 = 0$, the SHO equation for torsional oscillator is: $\frac{d^2\theta}{dt^2} = -\frac{\kappa}{I}\theta$

The rotational position θ of the torsional oscillator at instant t is given by: $\theta = \theta_{max}\sin(\omega t + \phi_i)$

where θ_{max} = max rotational displacement & $\omega = \sqrt{\frac{\kappa}{I}}$ angular frequency

For small rotational displacements, SHO E of pendulum is: $\frac{d^2\theta}{dt^2} = -\frac{m l_{cm} g}{I}\theta$ & $\omega = \sqrt{\frac{m l_{cm} g}{I}}$

where l_{cm} is the distance from the center of mass of the pendulum to the pivot

The period of a simple pendulum is $T = 2\pi\sqrt{\frac{l}{g}}$

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15.8- Damped Oscillations

In damped oscillation, the amplitude decreases over time due to energy dissipation. The cause of the dissipation is a damping force due to friction, air drag, or water drag.

At low speeds, the damping force $\vec{F}^{d}_{ao}$ tends to be proportional to the velocity of the object

$\vec{F}^{d}_{ao} = -b\vec{v}$ where b = damping coefficient (units: kg/sec)

For small damping, the position $x(t)$ of a damped spring is $x(t) = Ae^{-bt/2m}\sin(\omega_d t + \phi_i)$ and its angular frequency $\omega_d = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} = \sqrt{\omega^2 - (\frac{b}{2m})^2}$

The time constant T for a damped system is $T = \frac{m}{b}$. In one time constant, the energy of a damped simple harmonic oscillator is reduced by a factor of $1/e$. The amplitude $x_{max}(t)$ and energy $E(t)$ of a damped oscillation of initial amplitude A & initial energy E_0 decrease exponentially with time: $x_{max}(t) = Ae^{-t/2T}$, $E(t) = E_0 e^{-t/T}$

A damped oscillator that has a large quality factor Q will keep oscillating for many periods. $Q = 2\pi \frac{t}{T}$