Math 125D 3/6/24

More Review

Final Reminders:

1) Final is Saturday, 1:30-4:20 PM in KNE 110
   Cumulative, list of topics on Canvas
2) Form for req. makeups in email
3) Seating chart pref. form on Canvas.
4) Course evals!

Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- Turn off and stow away all cell phones, watches, pagers, music players, and other similar devices.
- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes.
- You can use only a Texas Instruments TI-30X IIS calculator. No other models are allowed.
- In order to receive credit, you must **show your work**. If you do not indicate the way in which you solved a problem, or if the work shown is incorrect or incomplete, you may get little or no credit for it, even if your answer is correct.
- You may use directly the integral formulas in the table below, without deriving them. **Show your work in evaluating any other integrals**, even if they are on your sheet of notes.

Table of Integration Formulas Constants of integration have been omitted.

1. $\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad (n \neq -1)$	**2.** $\int \frac{1}{x}\,dx = \ln\lvert x\rvert$
3. $\int e^x\,dx = e^x$	**4.** $\int b^x\,dx = \frac{b^x}{\ln b}$
5. $\int \sin x\,dx = -\cos x$	**6.** $\int \cos x\,dx = \sin x$
7. $\int \sec^2 x\,dx = \tan x$	**8.** $\int \csc^2 x\,dx = -\cot x$
9. $\int \sec x \tan x\,dx = \sec x$	**10.** $\int \csc x \cot x\,dx = -\csc x$
11. $\int \sec x\,dx = \ln\lvert \sec x + \tan x\rvert$	**12.** $\int \csc x\,dx = \ln\lvert \csc x - \cot x\rvert$
13. $\int \tan x\,dx = \ln\lvert \sec x\rvert$	**14.** $\int \cot x\,dx = \ln\lvert \sin x\rvert$
17. $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$	**18.** $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right), \quad a > 0$

- Place $\boxed{\text{a box around your answer}}$ to each question. Unless otherwise instructed, simplify your answers, but leave them in exact form (for example $\frac{\pi}{3}$ or $5\sqrt{3}$).
- All exam pages are double-sided except for this cover page and the last page. You may use the blank sides for extra room if needed but if you want us to grade these spare pages clearly **indicate in the problem area** that your work is on the back of the cover page or on the blank pages at the end of the exam.
- This exam has 10 problems on 10 pages. When the exam starts, make sure that your exam is complete. Good luck!

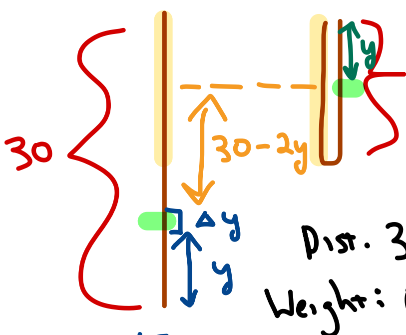
6. (10 points) A rope weighing 0.3 pounds per foot was tied to a robot and it was used to lower the robot into a 30-foot deep well.

   The robot will get out of the well by climbing up the rope at a constant speed, with the end of the rope still tied to the robot.

   At the beginning of the climb, the robot weighs 20 pounds, including fuel, but it will burn fuel at a constant rate and will lose 3 pounds of fuel during the climb.

   Compute the work that the robot will do in climbing up to the top of the well.

Rope	Robot

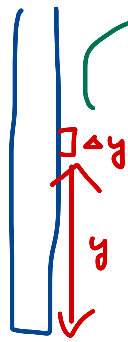


Dist. $30-2y$

Weight: $0.3\Delta y$

$$\int_0^{15} (30-2y)(0.3)\,dy$$

$= \dots$



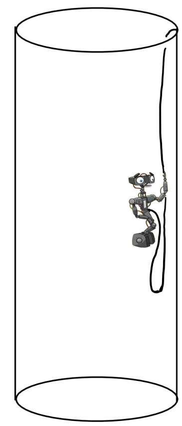
Work to move robot through this slice

Distance: Δy

Weight: $20 - \frac{1}{10}y$

$$\int_0^{30} \left(20 - \frac{1}{10}y\right) dy$$

$= \dots$



Robot loses 1 lb of fuel per 10 ft climbed

9. (10 points) Find the solution to the differential equation

$$y' = xy(y-1)$$

that satisfies the initial condition

$$y(0) = -1.$$

Give your solution in explicit form, $y = f(x)$.

$$\int \frac{dy}{y(y-1)} = \int x\,dx$$

$$\int \frac{1}{y(y-1)}\,dy = \int \left(\frac{-1}{y} + \frac{1}{y-1} \right) dy$$

Part. frac : $\frac{1}{y(y-1)} = \frac{A}{y} + \frac{B}{y-1}$

$$1 = A(y-1) + By$$

$y=1$: $1 = B$

$y=0$: $1 = -A$

$$-\ln|y| + \ln|y-1| = \frac{1}{2}x^2 + C$$

$$\ln\left|\frac{y-1}{y}\right| = \frac{1}{2}x^2 + C$$

$$\left|\frac{y-1}{y}\right| = e^{\frac{1}{2}x^2} e^{C}$$

$$\frac{y-1}{y} = \underbrace{\pm e^{C}}_{D} e^{\frac{1}{2}x^2}$$

$$1 - \frac{1}{y} = De^{\frac{1}{2}x^2}$$

$$\frac{1}{y} = 1 - De^{\frac{1}{2}x^2}$$

$$y = \frac{1}{1 - De^{\frac{1}{2}x^2}}$$

$x=0$ $y=-1$

$$-1 = \frac{1}{1-D} \qquad D = 2$$

$$\boxed{y = \frac{1}{1 - 2e^{\frac{1}{2}x^2}}}$$

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- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, or if the work shown is incorrect or incomplete, you may get little or no credit for it, even if your answer is correct.
- You may use directly the integral formulas # 1-18 in the table from section 7.5 of your textbook (posted on the departmental math 125 website), without deriving them. **Show your work in evaluating any other integrals, even if they are on your note sheet**.
- Place $\boxed{\text{a box around your answer}}$ to each question. Unless otherwise instructed, simplify your answers, but leave them in exact form (for example $\frac{\pi}{3}$ or $5\sqrt{3}$).
- If you need more room, use the backs of the pages and indicate that you have done so.
- This exam has 8 pages, in addition to this cover sheet. Make sure you have a complete exam.

Question	Points	Score
1	14	
2	14	
3	10	
4	14	
5	14	

Question	Points	Score
6	10	
7	10	
8	14	
Total	100	

4. (14 points) One model for air resistance predicts that a particular ball thrown straight up in the air will have velocity at t seconds given by:

$$v(t) = ce^{-t} - 10 \text{ meters/sec, for some constant } c,$$

where upward is considered positive velocity. Assume the ball is thrown straight upward starting from the ground with an initial velocity of 20 m/s.

(a) Find the formula for the height $h(t)$ of the ball after t seconds.

$v(t) = ce^{-t} - 10$

$v(0) = c - 10 = 20 \qquad c = 30$

some constant

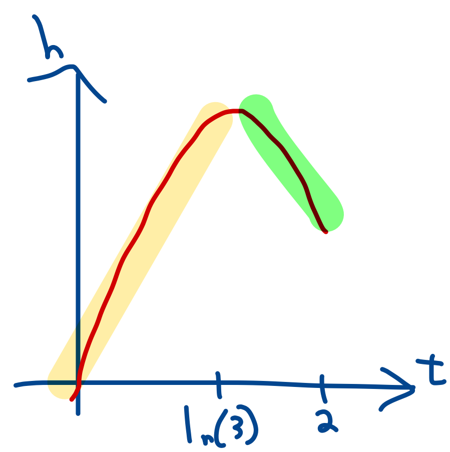
$h(t) = \int v(t)\,dt = \int (30e^{-t} - 10)\,dt = -30e^{-t} - 10t + D$

$h(0) = -30 + D = 0 \qquad D = 30 \qquad \boxed{h(t) = -30e^{-t} - 10t + 30}$

(b) Find the **total distance** traveled by the ball from $t = 0$ to $t = 2$ seconds. You may give your final answer as a decimal accurate to 3 digits after the decimal point (or leave in exact form).

When does $v(t) = 0$? $\quad 30e^{-t} - 10 = 0 \quad 30e^{-t} = 10 \quad e^{-t} = \frac{1}{3}$

$-t = \ln\left(\frac{1}{3}\right) \qquad t = \ln(3) = 1.$ something



Dist going up: $h(\ln(3)) - h(0)$

Dist. going down: $h(\ln(3)) - h(2)$

$h(\ln(3)) = -30\left(\frac{1}{3}\right) - 10\ln(3) + 30 = 20 - 10\ln(3)$

$h(2) = -30e^{-2} - 20 + 30 = 10 - 30e^{-2}$

$\text{Dist} = 2(20 - 10\ln 3) - (10 - 30e^{-2}) = \boxed{30 - 20\ln 3 + 30e^{-2} \text{ m}}$

5. (14 points) A particle is sliding down the curve $y = 10 - x^3$. At time $t = 0$ the particle starts at $(0, 10)$. The x-coordinate of the particle at time t is $x(t) = \frac{t}{3}$. Time is measured in seconds, distance in meters.

   Let $a(t)$ denote the arclength distance traveled by the particle along the curve in the first t seconds.

   (a) Set up an integral expression equal to $a(t)$. Do NOT attempt to evaluate it.

$y' = -3x^2$

$$a(t) = \int_0^{t/3} \sqrt{1 + (-3x^2)^2}\, dx$$

FTC: $\frac{d}{dt}\left(\int_a^t f(x)\,dx\right) = f(t)$

   (b) Calculate $a'(1)$. Include units.

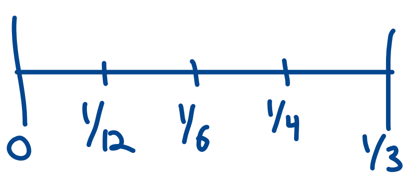
$$a'(t) = \sqrt{1 + \left(-3\left(\tfrac{t}{3}\right)^2\right)^2} \cdot \left(\tfrac{1}{3}\right)$$

$$= \sqrt{1 + \frac{t^4}{9}}\left(\frac{1}{3}\right) = \frac{\sqrt{9+t^4}}{\sqrt{9}}\,\frac{1}{3} = \frac{\sqrt{9+t^4}}{9}$$

$$a'(1) = \boxed{\frac{\sqrt{10}}{9}\ \text{m/s}}$$

   (c) Use Simpson's Rule with $n = 4$ subdivisions to approximate the value of $a(1)$. Show work, and give your answer correct to 3 decimal places.

Approximate $\int_0^{1/3} \sqrt{1 + 9x^4}\, dx$



$\Delta x = \frac{1}{12}$

$$\boxed{\frac{1}{36}\left(\sqrt{1} + 4\sqrt{1+9\left(\tfrac{1}{12}\right)^4} + 2\sqrt{1+9\left(\tfrac{1}{6}\right)^4} + 4\sqrt{1+9\left(\tfrac{1}{4}\right)^4} + \sqrt{1+9\left(\tfrac{1}{3}\right)^4}\right)}$$