Math 125D 2/21/24

Midterm 2 Review

DFEP #13 Solution:

Well, first off, $\displaystyle\int_0^1 \frac{dx}{\sqrt{x^3+5}}$ converges because it's just a regular definite integral. What about the rest? On the interval $[1,\infty)$, we know that $\sqrt{x^3+5} > \sqrt{x^3}$, so

$$\frac{1}{\sqrt{x^3+5}} < \frac{1}{x^{3/2}}.$$

Since $\displaystyle\int_1^\infty \frac{dx}{x^{3/2}}$ converges by the p-test, so does this.

Reminders: Midterm 2 tomorrow, in quiz section

Covers up through Chapter 7.8. ← Improper integrals

Bring: TI-30X IIS
One double-sided handwritten page of notes
Something to write with

Math 125C **Second Midterm** **Winter 2013**

Your Name

Your Signature

Student ID #

	Yuanlong		Chris	
Section (Thu.)	11:30	10:00	11:30	10:00
(circle one)	CA	CB	CC	CD

Problem	Total Points	Score
1	12	
2	12	
3	8	
4	8	
5	10	
Total	50	

- This exam is closed book. You may use one $8\frac{1}{2} \times 11$ sheet of notes.
- Do not share notes.
- Graphing calculators are not allowed.
- In order to receive credit, you must show your work. Do not do computations in your head. Instead, write them out on the exam paper.
- **Place a box around YOUR FINAL ANSWER to each question.**
- If you use a trial and error (or guess and check) method when an algebraic method is available, you will not receive full credit.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

5 (10 points) Determine if the improper integral $\int_{-1}^{0} \frac{e^{1/t}}{t^3}\,dt$ is convergent or divergent. If it is convergent, evaluate it.

$$\lim_{t\to 0^-}\int_{-1}^{t}\frac{e^{1/t}}{t^3}\,dt = \lim_{t\to 0^-}-\int_{-1}^{\frac{1}{t}} we^w\,dw = \lim_{t\to 0^-}\left(-we^w\Big]_{-1}^{1/t}+\int_{-1}^{1/t}e^w\,dw\right)$$

$w=\frac{1}{t}$ $\quad dw=\frac{-1}{t^2}\,dt$ $\quad u=w$ $\quad v=e^w$ $\quad du=dw$ $\quad dv=e^w\,dw$

$$=\lim_{t\to 0^-}\left(\left(-we^w+e^w\right)\Big]_{-1}^{1/t}\right)=\lim_{t\to 0^-}\left(\frac{-e^{1/t}}{t}+e^{1/t}-e^{-1}-e^{-1}\right)$$

($e^{1/t}\to -\infty$, $\frac{-e^{1/t}}{t}\to 0$)

$$\lim_{t\to 0^-}\frac{-e^{1/t}}{t}=\lim_{t\to 0^-}\frac{e^{1/t}\cdot\left(\frac{-1}{t^2}\right)}{1}=\lim_{t\to 0^-}\frac{-e^{1/t}}{t^2}$$

$\therefore$

Math 125 E and F **Midterm 2** **Burdzy, Autumn 2015**

Your Name

Your Signature

Student ID #

Circle quiz section and print TA's name:
EA EB EC ED FA FB FC FD

- Turn off all cell phones, pagers, radios, mp3 players, and other similar devices.
- This exam is closed book. You may use one $8\frac{1}{2}" \times 11"$ sheet of handwritten notes (both sides).
- You can use only Texas Instruments TI-30X calculator.
- Give your answers in exact form, not decimals, unless stated otherwise.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- **Check your work carefully.** We will award only limited partial credit.
- Place a box around your answer to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 6 pages, plus this cover sheet. Make sure that your exam is complete.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
Total	60	

2. (10 points) Determine whether the improper integral $\int_0^\infty e^{-x}\sin x\,dx$ is convergent. If it is convergent, find its value.

$$\lim_{t\to\infty}\int_0^t e^{-x}\sin x\,dx = \lim_{t\to\infty}\left(-e^{-x}\cos x\Big]_0^t - \int_0^t e^{-x}\cos x\,dx\right)$$

$u = e^{-x}$ $v = -\cos x$

$du = -e^{-x}dx$ $dv = \sin x\,dx$

$u = e^{-x}$ $v = \sin x$

$du = -e^{-x}dx$ $dv = \cos x\,dx$

$$\lim_{t\to\infty}\left(-e^{-x}\cos x - e^{-x}\sin x\Big]_0^t - \int_0^t e^{-x}\sin x\,dx\right)$$

$$\int_0^\infty e^{-x}\sin x\,dx = \frac{1}{2}\lim_{t\to\infty}\left(\left(-e^{-x}\cos x - e^{-x}\sin x\right)\Big]_0^t\right)$$

$$= \frac{1}{2}\lim_{t\to\infty}\left(-\underbrace{e^{-t}\cos t}_{0} - \underbrace{e^{-t}\sin t}_{0} + 1\right)$$

stays small

$\lim_{t\to\infty}\frac{\cos t}{e^t} = 0$ (e^t gets big)

$$= \boxed{\frac{1}{2}}$$

Math 125 B and C **Midterm 2** **Palmieri, Autumn 2006**

Your Name

Your Signature

Student ID #

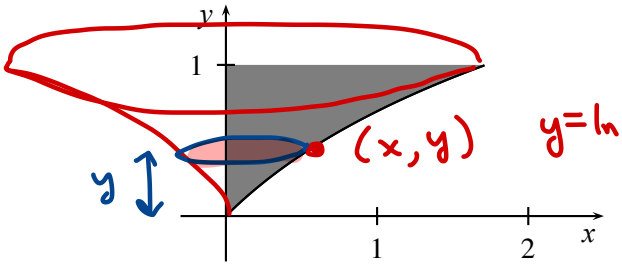
TA's Name and quiz section (circle):

Cady		Cruz		Jacobs	
BA	CB	BB	BC	CA	CC

- Turn off all cell phones, pagers, radios, mp3 players, and other similar devices.
- This exam is closed book. You may use one $8\frac{1}{2}" \times 11"$ sheet of handwritten notes (one side).
- Graphing calculators are not allowed.
- Give your answers in exact form, not decimals, except where indicated.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct. You may use any of the 20 integrals from the table on p. 506 of the text without deriving them. Show your work in evaluating any other integrals, even if they are on your note sheet.
- **Check your work carefully.** We will award only limited partial credit.
- Place $\boxed{\text{a box around your answer}}$ to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 4 pages, plus this cover sheet. Make sure that your exam is complete.

Question	Points	Score
1	16	
2	8	
3	8	
4	8	
5	10	
Total	50	

5. A portion of the graph of $y = \ln(x+1)$ between $x = 0$ and $x = e-1$ is rotated around the y-axis to form a container. The container is filled with water. Distance is measured in meters and the density of water is 1000 kg/m^3.



$y=\ln(x+1) \to e^y = x+1$

$x = e^y - 1$

(a) (6 points) Set up, but DO NOT EVALUATE, an integral to compute the work required to pump the water out over the side (which is at height $y = 1$).

Work for one slice π y: Dist = $(1-y)$

$F = 9.8$ [mass]

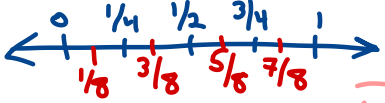
1000 [volume]

π [rad.]$^2 \Delta y$

$e^y - 1$

$$\int_0^1 9.8\left(1000\pi(e^y-1)^2\right)(1-y)\,dy$$

(b) (4 points) Use $n = 4$ subdivisions and the midpoint rule to approximate the value of the integral in part (a). Give an answer correct to at least two significant digits.



$\Delta y = \frac{1}{4}$

$$M_4 = (9800\pi)\frac{1}{4}\left((e^{1/8}-1)^2(7/8) + (e^{3/8}-1)^2(5/8) + (e^{5/8}-1)^2(3/8) + (e^{7/8}-1)^2(1/8)\right)$$

Math 125D **Second Midterm** **Autumn 2016**

Your Name

Your Signature

Student ID #

Quiz Section

PLEASE READ the DIRECTIONS below:

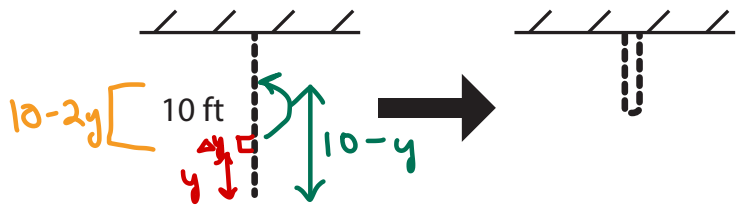
- Do not open the test until instructed to do so. This test has 5 problems on 5 pages. Once the test starts, please check that you have a complete exam.
- This exam is closed book. You may use one $8\frac{1}{2} \times 11$ page of handwritten notes. Do not share notes.
- Only a Ti-30x IIS calculator is allowed. Silence your cell phone and put it away.
- In order to receive credit, you **MUST SHOW YOUR WORK**. If we cannot tell how you are getting your answers, you may receive little or no credit, even if the answer happens to be correct.
- Simplify your answers as much as possible but leave them in exact form (e.g. $\pi\sqrt{2} + \frac{1}{2}$). Do not give decimal approximations, unless otherwise instructed.
- Place a **BOX** around $\boxed{\text{your final answer}}$ to each question.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- Read each question carefully, before and after answering it.

  Good luck!

Problem	Total Points	Score
1	8	
2	8	
3	10	
4	12	
5	12	
Total	50	

3. [**10 points**] A 10-ft chain weighs 25 lbs and hangs from a ceiling. Find the work done in lifting the lower end of the chain to the ceiling so that it's level with the upper end.

Show your work and how you set up any integrals you compute.



Work for slice = $25\left(\frac{\Delta y}{10}\right)(10-2y)$

force: $25\left(\frac{\Delta y}{10}\right)$ dist: $(10-2y)$

whatever fraction of the chain this is

$$\frac{25}{10}\int_0^5 (10-2y)\,dy$$

$$= 2.5\left(10y - y^2\right)\Big]_0^5$$

$$2.5(25) = \boxed{62.5 \text{ ft-lbs}}$$

4. Compute the following definite integrals. Simplify, but leave your answers in exact form.

(a) [**6 points**] $\int_0^{\pi/3} \tan^4(\theta)\, d\theta$ $= \int_0^{\pi/3} \tan^2\theta\left(\sec^2\theta - 1\right) d\theta$

$= \int_0^{\pi/3} \tan^2\theta\sec^2\theta\, d\theta - \int_0^{\pi/3} \tan^2\theta\, d\theta = \int_0^{\sqrt{3}} u^2\, du - \int_0^{\pi/3}\left(\sec^2\theta - 1\right) d\theta$

$\tan\left(\frac{\pi}{3}\right)$

$u = \tan\theta$
$du = \sec^2\theta\, d\theta$

$= \frac{u^3}{3}\Big]_0^{\sqrt{3}} - \left(\tan\theta - \theta\right)\Big]_0^{\pi/3}$

$= \sqrt{3} - \left(\sqrt{3} - \frac{\pi}{3}\right) = \boxed{\frac{\pi}{3}}$

(b) [**6 points**] $\int_0^{1/2} \arcsin(x)\, dx$