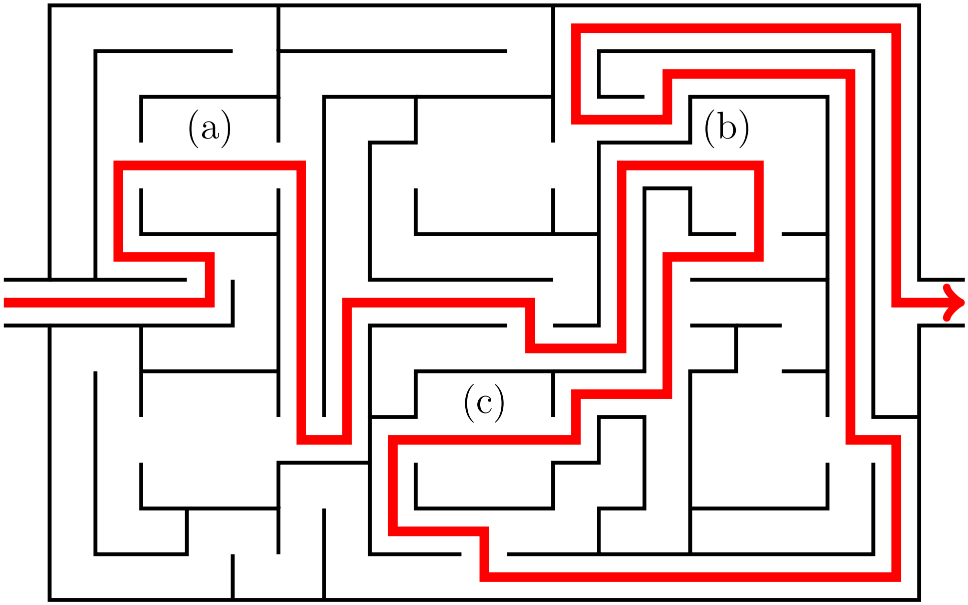
Math 125D 2/5/24

Chapters 7.2 & 7.3

DFEP #8 Solution:



(a) $\displaystyle\int \frac{\ln(x)}{x^3}dx$. Let $u = \ln(x)$, $dv = x^{-3}dx$, then this becomes $\displaystyle -\frac{\ln(x)}{2x^2} + \int \frac{1}{2x^3}dx$, or $\displaystyle -\frac{\ln(x)}{2x^2} - \frac{1}{4x^2} + C$.

(b) $\displaystyle\int \cos^5(x)\,dx = \int \cos(x)(1-\sin^2(x))^2\,dx = \int (1-u^2)^2\,du = \int (1-2u^2+u^4)\,du$. Integrate and resubstitute to get $\displaystyle \sin(x) - \frac{2}{3}\sin^3(x) + \frac{1}{5}\sin^5(x) + C$.

(c) $\displaystyle\int \frac{x}{e^{2x}}dx$. Let $u = x$, $dv = e^{-2x}dx$, so $du = dx$, $\displaystyle v = \frac{-1}{2}e^{-2x}\,dx$ and we get $\displaystyle \frac{-x}{2e^{2x}} + \int \frac{e^{-2x}}{2}\,dx = \frac{-x}{2e^{2x}} - \frac{1}{4e^{2x}} + C$.

DFEP #9: Monday, February 5th.

Let $\mathcal{R}$ be the region bounded by $y = 0$, $x = 0$, $x = 4$, and $y = \sqrt{x^2+6x+25}$. Compute the volume of the solid formed by revolving $\mathcal{R}$ around the y-axis.

How to integrate $\int \tan^m x \sec^n x \, dx$.

Main fact: ② $\tan^2 x + 1 = \sec^2 x$.

Ⓐ If n is even, reduce integral to a bunch of powers of $\tan x$, and one factor of $\sec^2 x \, dx$.

Then let $u = \tan x$, $du = \sec^2 x \, dx$

Ⓑ If m is odd, reduce integral to a bunch of powers of $\sec x$, and one factor of $\sec x \tan x \, dx$

Then let $u = \sec x$, $du = \sec x \tan x \, dx$

Ⓒ If n is odd <u>and</u> m is even, very tough!

Ex] Find $\int \tan^3 x \sec^5 x \, dx$

$$= \int \tan x (\tan^2 x) \sec^4 x \sec x \, dx$$

②

$$= \int \tan x (\sec^2 x - 1) \sec^4 x \sec x \, dx = \int (u^2 - 1) u^4 \, du = \int (u^6 - u^4) \, du = \frac{u^7}{7} - \frac{u^5}{5} + C$$

$u = \sec x$

$du = \sec x \tan x \, dx$

$$= \boxed{\frac{1}{7} \sec^7 x - \frac{1}{5} \sec^5 x + C}$$

Ex] Find $\int \frac{\sin^2 x}{\cos^4 x}\,dx = \int \left(\frac{\sin^2 x}{\cos^2 x}\right)\left(\frac{1}{\cos^2 x}\right) dx$

(Remember: $\tan x = \frac{\sin x}{\cos x}$

$\sec x = \frac{1}{\cos x}$)

$= \int \tan^2 x \sec^2 x\,dx = \int u^2\,du = \frac{1}{3}u^3 + C = \boxed{\frac{1}{3}\tan^3 x + C}$

$u = \tan x$
$du = \sec^2 x\,dx$

Ex] Find $\int \sec x\,dx$ mult. by $\frac{\sec x + \tan x}{\sec x + \tan x}$ $= \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x}\,dx$

$= \int \frac{1}{u}\,du = \ln|u| + C = \boxed{\ln|\sec x + \tan x| + C}$

$u = \sec x + \tan x$
$du = (\sec x \tan x + \sec^2 x)\,dx$

Ex] Find $\int \sec^3 x\,dx = \int \underbrace{\sec x}_{u}\,\underbrace{\sec^2 x\,dx}_{dv} = \sec x \tan x - \int \sec x \underbrace{\tan^2 x}_{\sec^2 x - 1}\,dx = \sec x \tan x - \int \sec^3 x\,dx + \int \sec x\,dx$

$u = \sec x$ $v = \tan x$
$du = \sec x \tan x\,dx$ $dv = \sec^2 x\,dx$

$2\int \sec^3 x\,dx = \sec x \tan x + \ln|\sec x + \tan x| + C$

$\int \sec^3 x\,dx = \boxed{\frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x| + C}$

Chapter 7.3: Trig Substitution

Main idea: Can use trig identities (& substitution) to simplify square roots of sums/differences of squares)

(Recall: $\sqrt{a^2+b^2} \neq a+b$)

Reminders:

$\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec^2 x$

$1 + \cot^2 x = \csc^2 x$

→ Allow us to combine <u>two</u> square terms into a single square term.

$\sin^2 x = \frac{1}{2}(1-\cos(2x))$ $\quad$ $\sin(2x) = 2\sin x \cos x$

$\cos^2 x = \frac{1}{2}(1+\cos(2x))$ $\quad$ $\cos(2x) = \cos^2 x - \sin^2 x$

Ex] Find $\int \sqrt{4-x^2}\, dx = \int \sqrt{4-4\sin^2\theta}\,(2\cos\theta)\,d\theta = \int \sqrt{4\cos^2\theta}\,(2\cos\theta)\,d\theta = \int 4\cos^2\theta\, d\theta$

If this said $4-4\sin^2\theta$, then it would simplify to $4\cos^2\theta$

Let $x = 2\sin\theta$

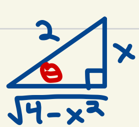
$dx = 2\cos\theta\, d\theta$

$\theta = \sin^{-1}\left(\frac{x}{2}\right)$

$= \int 2(1+\cos(2\theta))\,d\theta = 2\theta + \sin(2\theta) + C$

$= 2\theta + \overbrace{2\sin\theta}^{x}\cos\theta + C = 2\sin^{-1}\left(\frac{x}{2}\right) + x\underbrace{\cos\left(\sin^{-1}\left(\frac{x}{2}\right)\right)}_{\text{must simplify}} + C$

What's $\cos(\underbrace{\sin^{-1}(\tfrac{x}{2})}_{\theta})$?



$\cos\theta = \frac{\sqrt{4-x^2}}{2}$

Comp. Δ

$$= \boxed{2\sin^{-1}\left(\frac{x}{2}\right) + \frac{x\sqrt{4-x^2}}{2} + C}$$

General plan:

To integrate...	...substitute...	...so it becomes...	...which simplifies to...
$\sqrt{a^2-x^2}$	$x = a\sin\theta$	$\sqrt{a^2-a^2\sin^2\theta}$	$a\cdot\cos\theta$
$\sqrt{a^2+x^2}$	$x = a\tan\theta$	$\sqrt{a^2+a^2\tan^2\theta}$	$a\cdot\sec\theta$
$\sqrt{x^2-a^2}$	$x = a\cdot\sec\theta$	$\sqrt{a^2\sec^2\theta-a^2}$	$a\cdot\tan\theta$