Math 125D 1/26/24

Chapters 6.2 & 6.3

DFEP #5: Friday, January 26th.

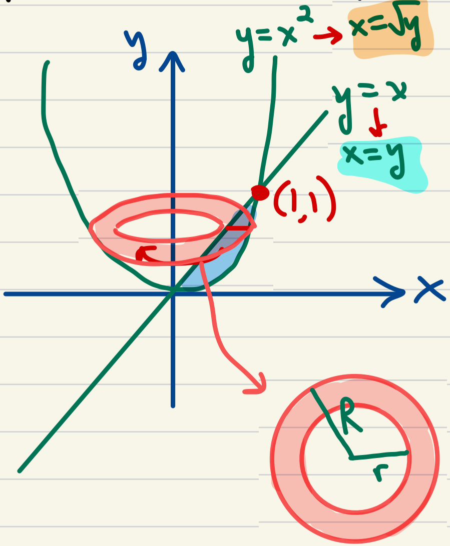
Compute the volume of the solid formed by rotating the region between $y = e^x$ and $y = \dfrac{x-1}{x}$ from $x = 1$ to $x = 4$ around the line $y = -1$.

Chapter 6.2 Continued

Ex] Let S be the solid formed by revolving the area btwn $y=x$ & $y=x^2$ around the y-axis.

Find the volume of S.

"washer method": disc method, but the cross-sections have holes.



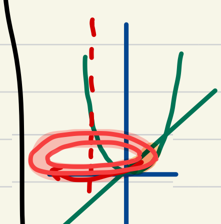
Integrate along y-axis, integrand is area of this ring.

$$\text{Volume} = \int_0^1 \Big(\underbrace{\pi\big(y - y^2\big)}_{\text{area of washer}}\Big)\,dy = (\text{compute this...})$$

$$\text{Area} = \pi\big(\underbrace{R^2}_{(\sqrt{y})^2} - \underbrace{r^2}_{(y)^2}\big)$$

Note $R^2 - r^2 \neq (R-r)^2$

What if I had revolved it around $x=-1$?



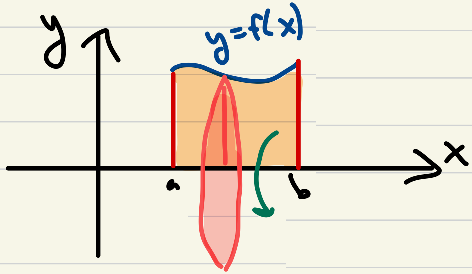
Same, but both radii are 1 unit bigger.

$$\int_0^1 \pi\Big(\big(\sqrt{y}+1\big)^2 - \big(y+1\big)^2\Big)\,dy$$

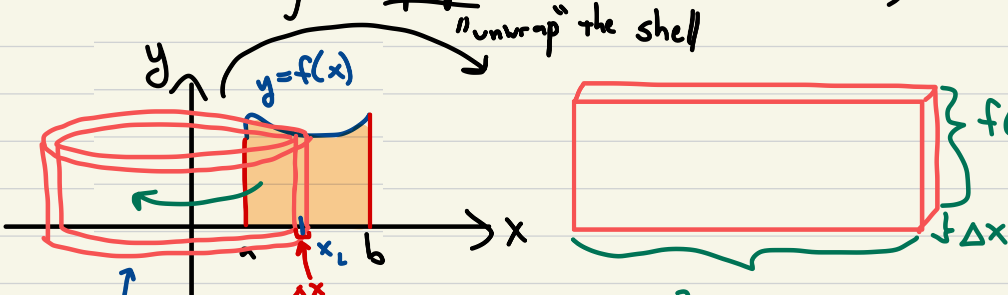
Chapter 6.3: The Shell Method

Previously, we found volumes by integrating parallel to the axis of rotation, and taking slices perpendicular to that axis.

$$\int_a^b \pi \left(f(x)\right)^2 dx$$



What if we integrate perp. to the axis of rotation and take slices parallel to that axis?



cylindrical shell

$$2\pi \underbrace{x_i}_{\text{radius of shell}}$$

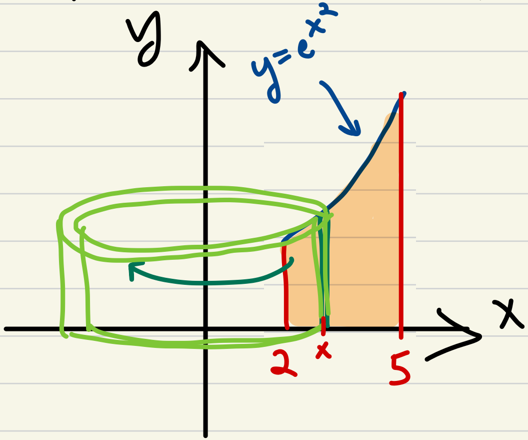
Total volume $= \sum_{i=1}^{n} 2\pi x_i f(x_i)\Delta x$

$\lim_{n\to\infty}$

$$\int_a^b 2\pi \underbrace{x}_{\text{radius of shell}} \underbrace{f(x)}_{\text{height of shell}}\, dx$$

Ex] Let R be the region below $y=e^{x^2}$ and above $y=0$, from $x=2$ to $x=5$.

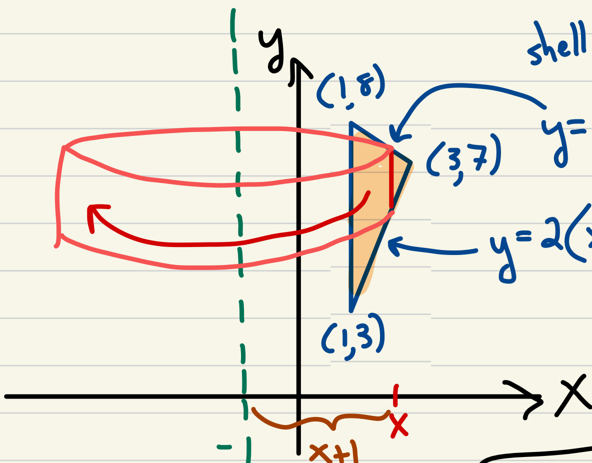
Find volume of solid formed by revolving R around y-axis.



Shell method: $\displaystyle\int_2^5 2\pi \underbrace{x}_{\text{rad}} \underbrace{e^{x^2}}_{\text{height}} dx$ = ... Compute w/ u-sub.

Ex] Let T be the triangle w/ vertices $(1,3)$, $(1,8)$, and $(3,7)$.

Find volume of solid formed by revolving T around $x=-1$.



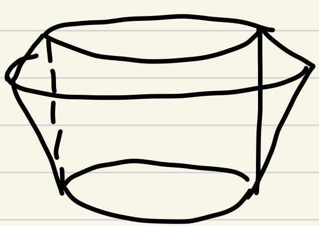
shell method

$y = \frac{-1}{2}(x-1)+8$

$y = 2(x-1)+3$

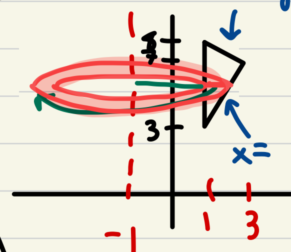
$$\int_1^3 2\pi \underbrace{(x+1)}_{\text{rad.}} \underbrace{\Big(\overbrace{\left(\tfrac{-1}{2}(x-1)+8\right)}^{\text{top}} - \overbrace{\left(2(x-1)+3\right)}^{\text{bottom}}\Big)}_{\text{height}}\, dx$$

compute...



Same, but washer?

$x = -2(y-8)+1$



$x = \frac{1}{2}(y-3)+1$

$$\int_3^7 \pi\left(\left(\tfrac{1}{2}(y-3)+1\right)^2 - 4\right) dy + \int_7^8 \pi\left(\left(-2(y-8)+1\right)^2 - 4\right) dy$$