Math 125D 1/24/24

Midterm 1 Review

DFEP #4 Solution:

What are the bounds on this region? $x^3 = ax$ means that $x = 0$, $x = \sqrt{a}$, or $x = -\sqrt{a}$. The region in the first quadrant runs from 0 to $\sqrt{a}$, and over that interval, $ax \geq x^3$

So the area in question is

$$\int_0^{\sqrt{a}} (ax - x^3)\, dx = \frac{ax^2}{2} - \frac{x^4}{4}\Big]_0^{\sqrt{a}} = \frac{a^2}{4} = 5$$

So $a = \sqrt{20}$.

Reminders:

- Midterm 1 is tomorrow, in quiz section. (80 minutes)
- 5 pages, 6 problems.
- Covers up through Chapter 6.1.
- Bring: TI-30X IIS
  One double-sided handwritten page of notes
  Something to write with

Graphs to be familiar with:

Lines, parabolas, polynomials in general

Exponential & logarithmic functions

Rational functions

Trig & inverse trig functions

Circles & semicircles

(And how to shift & strech these)

Math 125, Sections D and E, Midterm I

January 28, 2016

Name____

TA/Section______________________________________

Instructions.

- There are 4 questions. The exam is out of 40 points.
- You are allowed to use one page of notes written only on one side of the sheet in your own handwriting. Hand in you note sheet with your exam.
- You can use a Ti-30x IIS calculator. Put away all other electronic devices.
- For your integrals you may use the following formulas. Anything else must be justified by your work.

$$\int x^n\ dx = \frac{x^{n+1}}{n+1} + C, n \neq -1 \qquad \int e^x\ dx = e^x + C \qquad \int \frac{1}{x}\ dx = \ln|x| + C$$

$$\int \sin x\ dx = -\cos x + C \qquad \int \cos x\ dx = \sin x + C \qquad \int \sec^2 x\ dx = \tan x + C$$

$$\int \csc x \cot x\ dx = -\csc x + C \qquad \int \sec x \tan x\ dx = \sec x + C = \qquad \int \csc^2 x\ dx = -\cot x + C$$

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C \qquad \int \frac{1}{1+x^2} dx = \tan^{-1} x + C$$

- **Show your work**. If I cannot read or follow your work, I cannot grade it. You may not get full credit for a right answer if your answer is not justified by your work. If you continue at the back of a page, make a note for me.

Question	points
1	
2	
3	
4	
Total	

1. Evaluate the following integrals.

(a) (3 points) $\int \frac{x+3}{\sqrt{x+2}}dx$ $= \int \frac{u+1}{\sqrt{u}} du = \int \left(\frac{u}{\sqrt{u}} + \frac{1}{\sqrt{u}}\right) du$

$u = x+2$
$du = dx$

$= \int \left(\sqrt{u} + \frac{1}{\sqrt{u}}\right) du$

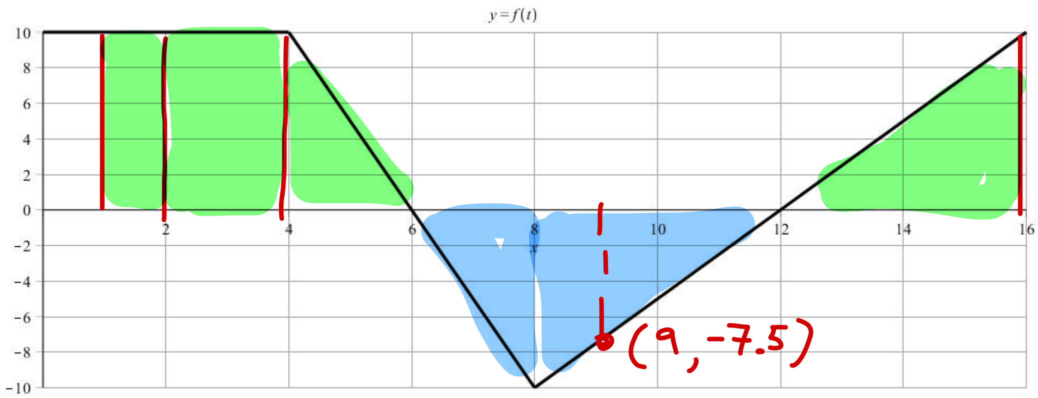
$= \frac{2u^{3/2}}{3} + 2u^{1/2} + C$

$= \boxed{\frac{2}{3}(x+2)^{3/2} + 2(x+2)^{1/2} + C}$

(b) (3 points) $\int_0^1 x(5x^2+1)^2 dx$

(c) (3 points) $\int_0^5 |x-3|\, dx$

2. Define $g(x) = \int_2^{x^2} f(t)dt$ where the graph of $f(t)$ is given below.



(a) (2 points) Compute $g(2)$.

$\int_a^b \sim + \int_b^c \sim = \int_a^c \sim$

$\left(\int_a^c \sim\right) - \left(\int_a^b \sim\right) = \int_b^c \sim$

$\int_2^4 f(t)dt = 2 \cdot 10 = \boxed{20}$

(b) (2 points) Express $g(4) - g(1)$ as a definite integral and compute its value.

$g(4) - g(1) = \int_2^{16} f(t)dt - \int_2^1 f(t)dt = \boxed{\int_1^{16} f(t)dt} = 3 \cdot 10 = \boxed{30}$

(c) (4 points) Compute $g'(3)$.

$\frac{d}{dx}\left(\int_2^{x^2} f(t)dt\right)$ when $x = 3$

(FTC) ↓

Need chain rule: $f(x^2) \cdot (2x)$

When $x=3$: $f(9) \cdot 6 = \boxed{(-7.5)(6)}$

(d) (2 points) Compute $g''(1)$.

$g'(x) = f(x^2)(2x)$

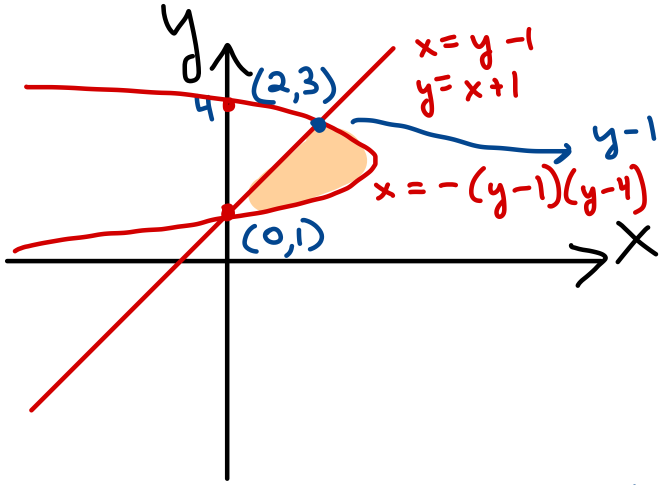
↘ product rule

$g''(x) = f'(x^2)(2x)^2 + f(x^2)(2)$

$g''(1) = f'(1)(2)^2 + f(1)(2)$

$= 4\underbrace{f'(1)}_{0} + 2\underbrace{f(1)}_{10} = \boxed{20}$

3. (10 points) Find the area of the region to the left of the parabola $x = -(y-1)(y-4)$ and below the line $x = y - 1$. Include a picture of the region.



$$y - 1 = -(y-1)(y-4)$$

$$y-1 + (y-1)(y-4) = 0$$

$$(y-1)(1 + y - 4) = 0$$

$$(y-1)(y-3) = 0$$

Intersect when $y = 1$ or 3

$$\int_1^3 \left(-(y-1)(y-4) - (y-1)\right) dy = \int_1^3 (y-1)(-(y-4)-1)\,dy$$

$$= \int_1^3 (y-1)(3-y)\,dy = \int_1^3 (-y^2 + 4y - 3)\,dy \quad \text{etc...}$$

Name____

Math 125 **First Midterm** **8:30 Jan. 31, 2019**

(6 problems, 80 minutes, 100 points, 1 sheet of notes but no calculator, no cellphone, no watch permitted)

Please show all your work clearly, and box your final answers. Leave your answers in exact form, but make any obvious simplifications for full credit. Cross out any work that you don't want us to consider. If you run out of space on the problem page, use the back of that page.

1. (15 points) Let

$$f(x) = \frac{1}{4x-3} \qquad \text{and} \qquad g(x) = \frac{1}{x^2}.$$

(a) Find the points of intersection of the two curves. Between these two points, which curve is above the other?

(b) Find the area between the two curves.

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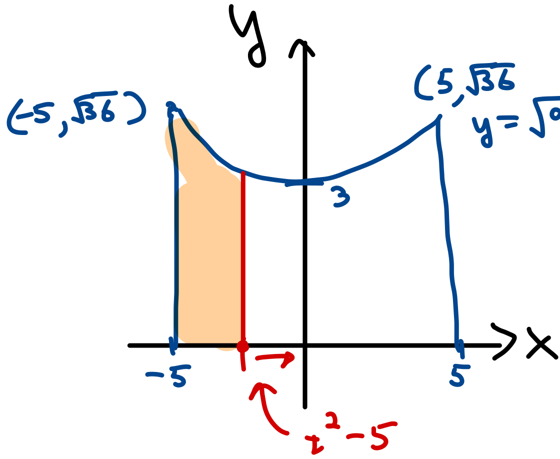
4. (15 points) The curve $y = \sqrt{9 + x^2}$ for $-5 \le x \le 5$ is the top of a stage opening; the bottom of the stage is the x-axis. The units are meters. At time $t = 0$ the opening is covered, but at that moment a curtain starts to uncover the stage starting from $x = -5$ and moving to the right. The curtain starts from rest and accelerates at 2 m/sec^2.

(a) Find the equation for the position of the curtain at time t, and then write an integral for the area that's been uncovered at time t.

(b) Find the rate (in square meters per second) at which the uncovered area is increasing when $t = 3$ sec.



$a(t) = 2$

$v(t) = 2t + \cancel{C}$ $\quad v(0) = 0 \quad C = 0$

$s(t) = t^2 + \underbrace{C_1}_{-5} \quad s(0) = -5$

a) $\boxed{\text{Position} = t^2 - 5 \text{ m} \qquad \text{Area} = \int_{-5}^{t^2-5} \sqrt{9+x^2}\, dx \text{ m}^2}$

b) $\frac{d}{dt}\left(\int_{-5}^{t^2-5} \sqrt{9+x^2}\, dx \right) = \sqrt{9 + (t^2-5)^2}\ (2t)$

$t = 3$: $\sqrt{9 + (4)^2}\ (6) = \boxed{30 \text{ m}^2/\text{sec}}$

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