Math 125D 1/19/24

Chapters 5.5 & 6.1

DFEP #2 Solution:

Okay, we need the second derivative of $f(x) = \displaystyle\int_{\sin(2x)}^{3x} e^{t^2}\,dt$.

Let $g(x) = \int_0^x e^{t^2}\,dt$. According to the fundamental theorem of calculus, $g'(x) = e^{x^2}$. Furthermore, $f(x) = g(3x) - g(\sin(2x))$. So:

$$f'(x) = 3g'(3x) - 2\cos(2x)g'(\sin(2x)) = 3e^{(3x)^2} - 2\cos(2x)e^{\sin^2(2x)}$$

Differentiating again gives:

$$f''(x) = 54xe^{(3x)^2} + 4\sin(2x)e^{\sin^2(2x)} - 8\cos^2(2x)\sin(2x)e^{\sin^2(2x)}$$

And therefore $f''\left(\frac{\pi}{2}\right) = 54\left(\frac{\pi}{2}\right)e^{(3\pi/2)^2} > 0$, so the function is concave up.

DFEP #3: Friday, January 19th.

Your train leaves New York for Philadelphia at 9:00 AM at a speed of 100 miles per hour. Seated next to you on the train is a man staring at a page of tricky integrals. Solve the integrals for him.

(a) $\displaystyle\int \frac{1}{x\ln(x^2)}\,dx$

(b) $\displaystyle\int e^{e^x+x}\,dx$

(c) $\displaystyle\int_0^3 x^5\sqrt[3]{x^2-16}\,dx$

Chapter 5.5 Continued

Ex] $\frac{1}{6}\int 6x\sqrt{\sin(3x^2)}\cos(3x^2)\,dx = \frac{1}{6}\int\sqrt{\sin(u)}\cos(u)\,du = \frac{1}{6}\int\sqrt{w}\,dw = \frac{1}{6}\frac{2w^{3/2}}{3} + C$

$u = 3x^2$

$du = 6x\,dx$

$w = \sin(u)$

$dw = \cos(u)\,du$

$= \frac{1}{9}(\sin(u))^{3/2} + C$

$\boxed{= \frac{1}{9}(\sin(3x^2))^{3/2} + C}$

OR: $u = \sin(3x^2)$

$du = \cos(3x^2)\,6x\,dx$

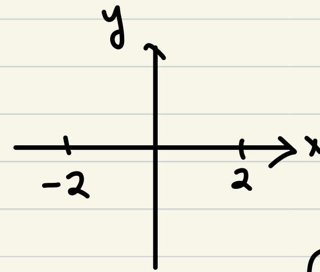
$\downarrow$

only 1 u-sub necessary

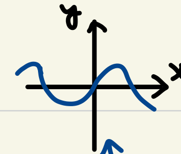
Symmetry of Integrals:

odd + odd = odd
even + even = even
odd · odd = even
odd · even = odd
even · even = even

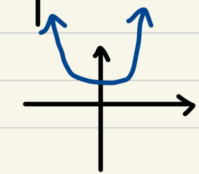
Compute $\displaystyle\int_{-2}^{2} \frac{\sin(x)}{\underbrace{8+9x^2+3x^{14}}_{f(x)}}\,dx$



Sin x:



$8+9x^2+3x^{14}$



How does $f(-x)$ compare to $f(x)$?

$$\underbrace{f(-x) = -f(x)}$$

This integrand is an "odd function"

So $\displaystyle\int_{-a}^{a} f(x)\,dx = 0$ if f is an odd function.

$f(x)$ is an even function if $f(-x) = f(x)$

(i.e. it has mirror symmetry across y-axis)

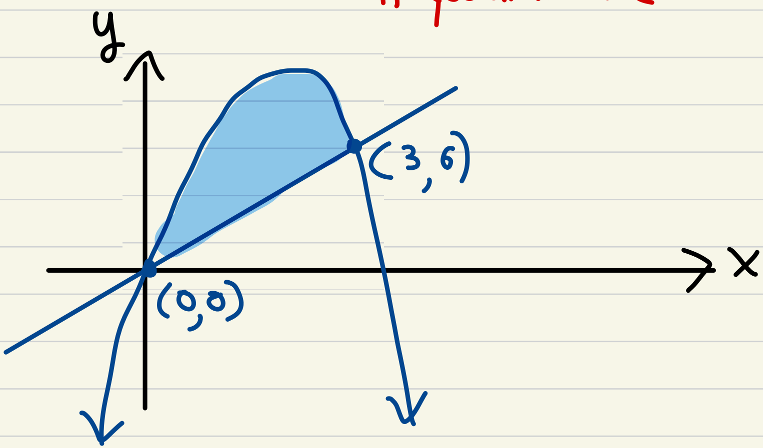
$$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$$

if f is an even function

Chapter 6.1: Areas Between Curves

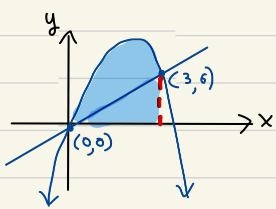
Ex] Find the area $\underbrace{\text{bounded by}}_{\text{If you draw these curves, what area do they enclose?}}$ $\overbrace{y=2x \text{ and } y=5x-x^2}^{\text{where do these intersect?}}$.

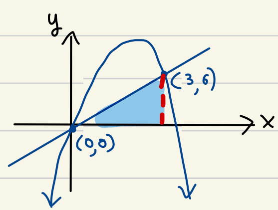
$$2x = 5x - x^2$$
$$x^2 - 3x = 0$$
$$\underbrace{x(x-3)=0}_{x=0 \ \& \ x=3}$$



General idea: the area under $y=f(x)$ and above $y=g(x)$ on $[a,b]$ is

$$\int_a^b \big(f(x)-g(x)\big)\,dx$$



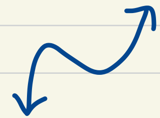


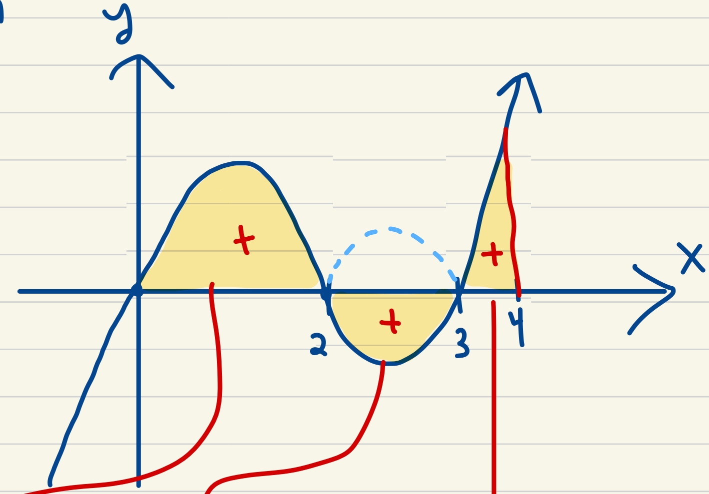
$$= \int_0^3 (5x-x^2)\,dx - \int_0^3 2x\,dx = \int_0^3 \big((5x-x^2)-2x\big)\,dx\,dx$$

$$= \int_0^3 (3x-x^2)\,dx = \left(\frac{3}{2}x^2 - \frac{1}{3}x^3\right)\Bigg]_0^3 = \frac{27}{2} - 9 = \boxed{4.5}$$

Ex] Find $\int_0^4 \left| x^3 - 5x^2 + 6x \right| dx$

What does $y = x^3 - 5x^2 + 6x$ look like?





Where does $x^3 - 5x^2 + 6x = 0$?

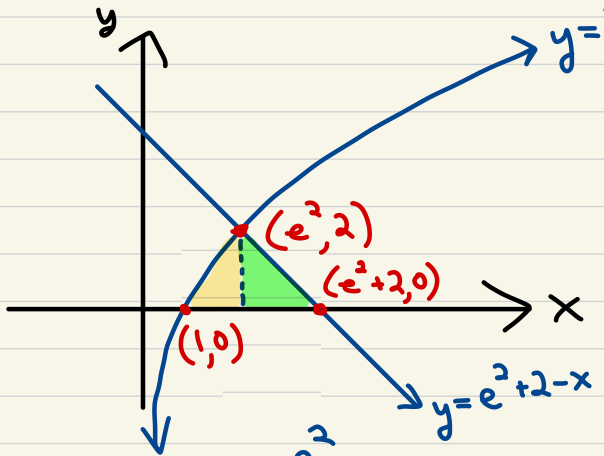
$$x(x^2 - 5x + 6) = 0$$

$$x(x-3)(x-2) = 0$$

$$x = 0,\ 2,\ \&\ 3$$

$$\underbrace{\int_0^2 (x^3 - 5x^2 + 6x)\,dx - \int_2^3 (x^3 - 5x^2 + 6x)\,dx + \int_3^4 (x^3 - 5x^2 + 6x)\,dx}_{\text{compute this}}$$

Ex] Find area bounded by $y=\ln(x)$, $y=e^2+2-x$, and the x-axis.



where does $y=\ln(x)$ intersect $y=\underbrace{e^2+2-x}_{\text{this is a hint!}}$?

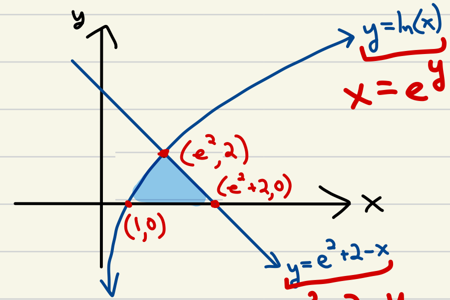
Plug in e^2:

$\ln(e^2)=2$ $\quad$ $e^2+2-e^2=2$

Left piece: $\displaystyle\int_1^{e^2} \ln(x)\,dx$

Right piece: $\displaystyle\int_{e^2}^{e^2+2} (e^2+2-x)\,dx$

Better way | Do this sideways:



$x=e^y$

$x=e^2+2-y$

In terms of y:

The "top function" is the one with the bigger x-co

$$\int_0^2 \left((e^2+2-y)-e^y\right)dy$$