Math 125D 1/17/24

Chapter 5.5

DFEP #1 Solution:

(a) We want to write $\lim_{n \to \infty} \sum_{i=1}^{n} \cos\left(\frac{i\pi}{3n}\right) \frac{\pi}{3n}$ as an integral. Recall that using a limit of right-hand Riemann sums, we have $\int_{a}^{b} f(x) dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(a+i\Delta x)\Delta x$, where $\Delta x = \frac{b-a}{n}$. Stare at this limit a bit and you can see it's what you get when $a = 0, b = \frac{\pi}{3}$,

Stare at this limit a bit and you can see it's what you get when $a = 0, b = \frac{\pi}{3}$ and $f(x) = \cos(x)$.

So it's

$$\int_0^{\pi/3} \cos(x) \, dx = \sin(x) \Big]_0^{\pi/3} = \sin\left(\frac{\pi}{3}\right) - \sin(0) = \frac{\sqrt{3}}{2}.$$

(b) Hey, I warned you, this one is tough. First, a little algebra:

$$\lim_{n \to \infty} \sum_{i=0}^{n-1} \frac{1}{\sqrt{n^2 - \frac{i^2}{4}}} = \lim_{n \to \infty} \sum_{i=0}^{n-1} \frac{1}{n\sqrt{1 - \left(\frac{i}{2n}\right)^2}}$$

That $\frac{i}{2n}$ sure looks like it wants to be x_i , which means Δx should be $\frac{1}{2n}$. So rewrite it as:

$$\lim_{n \to \infty} \sum_{i=0}^{n-1} \frac{2}{\sqrt{1 - \left(\frac{i}{2n}\right)^2}} \cdot \frac{1}{2n}$$

This is the limit of left-hand Riemann sums for the integral

$$\int_0^{\frac{1}{2}} \frac{2}{\sqrt{1-x^2}} \, dx = 2 \arcsin(1/2) - 2 \arcsin(0) = \frac{\pi}{3}$$

DFEP #2: Wednesday, January 17th.

Consider the function $f(x) = \int_{\sin(2x)}^{3x} e^{t^2} dt$.

Is f(x) concave up or concave down at $x = \frac{\pi}{2}$?

Chapter S.5: U-substitution
Exhibit at is
$$\boxed{2xe^{\frac{x^{2}}{2}}dx}$$
? It $e^{x} + C$. $Uhy? \frac{d}{dx}(e^{x^{2}}) = e^{x}(2x)$
Note: $\frac{d}{dx}(x^{2}) = 2x$
How can be notice situations where we can "undo" the chain rule?
Iden $\frac{d}{dx}(f(g(x))) = f'(g(x))g'(x)$. Look for (a) A function $g(x)$ there $\frac{d}{dx}(f(g(x))) = f'(g(x))g'(x)$. Look for (a) A function $g(x)$ there $\frac{d}{dx}(f(g(x))) = f'(g(x))g'(x)$. Look for (a) A function $g(x)$ there $\frac{d}{dx}(f(g(x))) = f'(g(x))g'(x)$. Look for (a) A function $g(x)$ there $\frac{d}{dx}(f(g(x))) = f'(g(x))g'(x)$. Look for (a) A function $g(x)$ there $\frac{d}{dx}(f(g(x))) = f'(g(x))g'(x)$. Look for (a) A function $g(x)$ there $\frac{d}{dx}(f(g(x))) = f'(g(x))g'(x)$. Look for (b) A function $g'(x)$.
Process: Take that "inner function $g(x)$ and call it u .
If $g(x)=u$, then $g'(x)=\frac{du}{dx} \rightarrow du = g'(x)dx$
Replace $\frac{du}{dx}$ 'x's in the integral with 'u's using these two equations
Convert final answer back into terms of x_{0} .

Ex) Find
$$\int \sqrt{3} x \cos x \, dx$$
. = $\int \sqrt{4} \, du = \int \sqrt{4} \, du = \frac{2u^{3/2}}{3} + C = \frac{2(\sin x)^{3/2}}{3} + C$
 $u = \sin x - \frac{3}{4} + C = \frac{3}{4} +$

-

$$\begin{array}{c} \text{Ex} \ F_{\text{ind}} \ \int 4 \sqrt{4x+1} \ dx \ = \ \int \sqrt{u} \ du \ = \ \frac{2 u^{3/2}}{3} \ + \ C \ = \ \frac{2 \left(4x+1\right)^{3/2}}{3} \ + \ C \ \end{array}$$

$$du = 4dx$$

$$Ex \int Find \int cos(\pi x) dx = \int \frac{1}{\pi} cos(u) du = \frac{1}{\pi} sin(u) + C = \frac{1}{\pi} sin(\pi x) + C$$

$$u = \pi r x$$

$$or^{2} \int cos(\pi x) \pi dx = \frac{1}{\pi} \int cos(u) du = \frac{1}{\pi} sin(u) + C \dots$$

$$du = \pi dx$$

$$u = \pi r dx$$

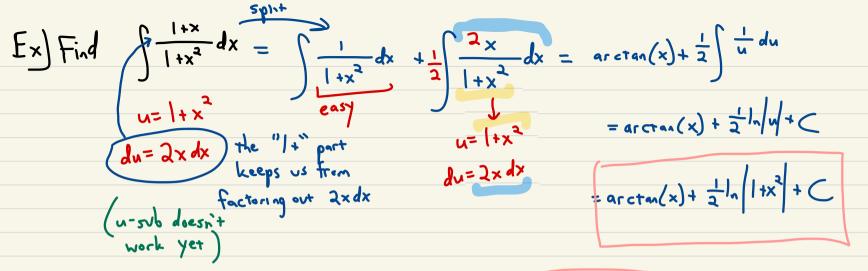
$$du = \pi dx$$

$$du = \pi dx$$

$$du = \pi dx$$

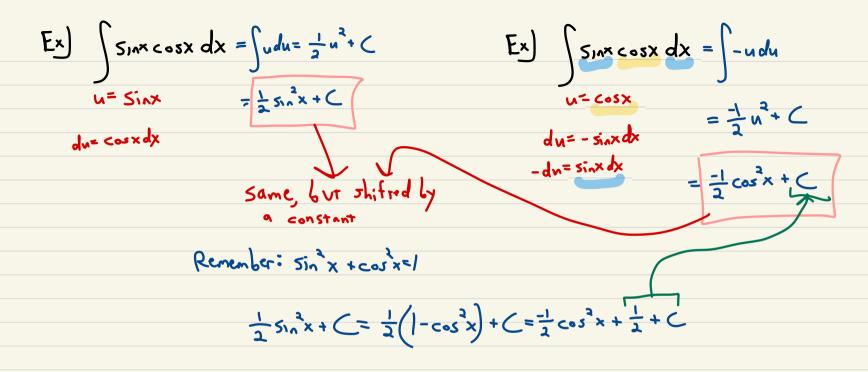
$$du = \pi dx$$

$$du = r dx$$



$$Ex)\int tan x \, dx = \int \frac{5nx}{\cos x} \, dx = \int \frac{-1}{u} \, du = -\ln|u| + C = -\ln|\cos x| + C$$

du= - sinx dx - du= sinx dx



How does u-sub work of def. integrals?
Ex]
$$\int_{0}^{1} \sin(e^{x}) e^{x} dx$$
. $du = e^{x} dx$
Method (1)
Remember that your bounds are in terms of x not a. $\int \sin(u) du = -\cos(u)$
Write antid. In terms of x before plugging in? $-\cos(e^{x})$] = $-\cos(e^{1}) + \cos(e^{2})$
Method (2)
Change bounds to be in terms of u, not x.
 $\int_{1}^{1} \sin(e^{x}) e^{x} dx = \int_{1}^{1} \sin(u) = -\cos(e^{1}) + \cos(e^{1})$
 $u = e^{x} dv = e^{x} dx$
Bounds: $x = 1$
 $u = e^{x} = 0$
 $u = e^{x} = 1$