Math 125D 1/10/24

Chapters 5.2 & 5.3

Chapter 5.2, Continued

Ex] Write $\displaystyle\int_1^3 \overbrace{(x^2+2x)}^{f(x)}\,dx$ as a limit of Riemann sums.

$$\overbrace{\lim_{n\to\infty}\sum_{i=1}^{n} \underbrace{\left(\frac{2}{n}\right)}_{\Delta x}\underbrace{\left(\left(1+\frac{2i}{n}\right)^2+2\left(1+\frac{2i}{n}\right)\right)}_{f(x_i)}}^{R_n}$$

$$\Delta x = \frac{b-a}{n} = \frac{3-1}{n} = \frac{2}{n}$$

$$x_i = a + i\Delta x = 1 + i\left(\frac{2}{n}\right) = 1+\frac{2i}{n}$$

Ex] Write $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n} \underbrace{\left(\frac{7}{n}\right)}_{\frac{b-a}{n}} \sin\left(\left(\overbrace{\underbrace{2}_{a}+\underbrace{\frac{7i}{n}}_{i\Delta x}}^{x_i}\right)^2\right)$ as a definite integral.

$$\int_2^9 \sin(x^2)\,dx$$

$\to b-a=7$, so $b=a+7=9$

Chapter 5.3: Fundamental Theorem of Calculus

actually 2 theorems

Part 1

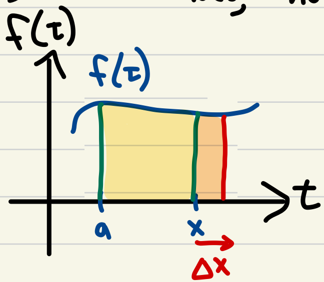
Suppose $g(x)=\int_a^x f(t)\,dt$. What's $g'(x)$?

constant

some other function

To find $g'(x)$:

We want to know, as x increases, how fast does $g(x)$ increase?

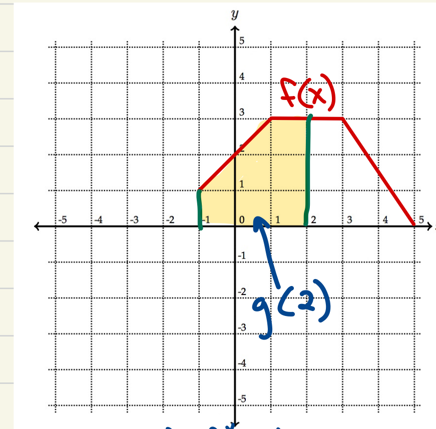


when Δx is very small, this extra area is almost a rectangle, with height $f(x)$ and width Δx

So $g'(x)=\lim\limits_{\Delta x\to 0}\dfrac{\text{Tiny extra area}}{\Delta x}=\lim\limits_{\Delta x\to 0}\dfrac{f(x)\cancel{\Delta x}}{\cancel{\Delta x}}=f(x)$

$$\boxed{\text{FTC Part 1: } \frac{d}{dx}\left(\int_a^x f(t)\,dt\right)=f(x)}$$

constant



Let $g(x)=\int_{-1}^{x} f(t)\,dt$.

What's $g(2)$? $\int_{-1}^{2} f(t)\,dt = 7$

$g(-1)=0$

$g(5)=13$

$g'(x)=\lim\limits_{h\to 0}\dfrac{g(x+h)-g(x)}{h}$

We just learned that $\frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x)$. That means $\int_a^x f(t)\,dt$ is an antideriv. of $f(x)$.

Remember: any two antideriv's of a continuous function differ only by a constant

So, if $F(x)$ is an antider of $f(x)$, then $\int_a^x f(t)\,dt = F(x) + C$. What's C?

plug in $x=a$

$$\underbrace{\int_a^a f(t)\,dt}_{0} = F(a) + C \rightarrow 0 = F(a) + C \rightarrow C = -F(a)$$

We get $\int_a^x f(t)\,dt = F(x) - F(a) \xrightarrow{\text{change var names}} \int_a^b f(x)\,dx = F(b) - F(a)$

FTC Part II: If f is continuous on $[a,b]$ and F is an antideriv of f,

then: $$\boxed{\int_a^b f(x)\,dx = F(b) - F(a)}$$

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Ex] Compute $\int_1^3 (x^2+2x)\,dx$. Need antider. of x^2+2x. Use $F(x) = \frac{x^3}{3} + x^2$.

FTC says: $\int_1^3 (x^2+2x)\,dx = \underbrace{F(3)-F(1)}_{} = \left(\frac{3^3}{3}+3^2\right) - \left(\frac{1^3}{3}+1^2\right) = 18 - \frac{4}{3} = \boxed{\frac{50}{3}}$

other notation: $F(x)\Big]_1^3$, $\Big[F(x)\Big]_1^3$, or $F(x)\Big|_1^3$

Another way to write this: $\int_1^3 (x^2+2x)\,dx = \left(\frac{x^3}{3}+x^2\right)\Big]_1^3 = \left(\frac{3^3}{3}+3^2\right) - \left(\frac{1^3}{3}+1^2\right) = \frac{50}{3}$

Ex] Compute $\int_1^4 \frac{3}{1+x^2}\,dx = 3\arctan(x)\Big]_1^4 = \boxed{3\arctan(4) - \frac{3\pi}{4}}$

(Or: $= \left(3\arctan(x) + 17\right)\Big]_1^4 = \left(3\arctan(4) \cancel{+17}\right) - \left(3\arctan(1) \cancel{+17}\right)$)

Ex] $\int_0^{2\pi/3} \underbrace{4\sec^2(x)}\,dx = 4\tan(x)\Big]_0^{2\pi/3} = 4\tan\left(\frac{2\pi}{3}\right) - 4\tan(0) = -4\sqrt{3}$

discontinuity at $x = \frac{\pi}{2}$! Can't use FTC.

Can't solve this yet!