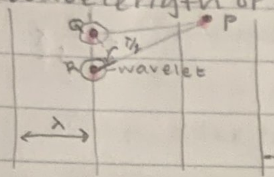
Chapter 34 Textbook Notes - Wave and Particle optics

34.1 - Diffraction of Light

- Diffraction is observed in light but only when the width of the opening through which the light passes is not much greater in size than the wavelength of the light

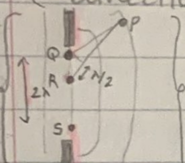
For when the opening is much larger than the window (no diffraction)



Points Q & R are on the same wavefront, so the wavelets are in phase. Since their distance to P differs by $\frac{\lambda}{2}$, the waves are 180° out of phase and therefore interfere destructively at P

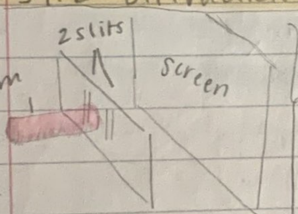
- The intensity (power/area) of a planar wave is uniform as the wave propagates forward because the fields from individual wavelets cancel in any outward direction and reinforce only in the direction of propagation.

For small openings

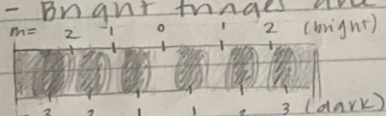


Here, Q & R cancel out (destructive interference at P) but the barrier means that there is no point to cancel out S. This creates diffraction of light waves.

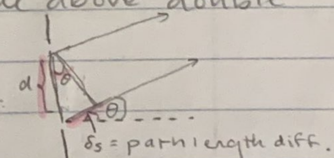
34.2 - Diffraction Gratings



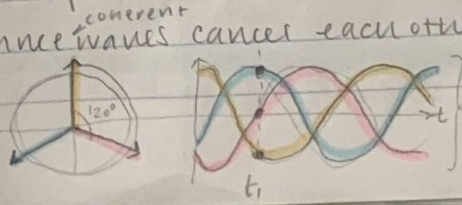
- The 2 slits act as 2 coherent point sources
  - Constructive interference where crests overlap
  - Destructive interference between crest overlaps
  - Creates a pattern of interference fringes (bright and dark bands) on the screen
  - Bright fringes are labeled by fringe order m (central m=0)



interference pattern caused by the above double slit setup



- θ = angle between nearly parallel rays & normal to barrier
- $\delta_s = d\sin\theta$
  - When $\delta_s = m\lambda$, constructive interference occurs ($d\sin\theta_m = \pm m\lambda$) = bright fringe
  - When $\delta_s = \pm(n-\frac{1}{2})\lambda$, destructive intf. occurs ($d\sin\theta_n = \pm(n-\frac{1}{2})\lambda$) = dark fringe
- The location of bright fringes does not depend on the number of slits as long as the separation d between adjacent slits is the same for all slits.
- Three coherent waves cancel each other out when each is 1/3 out of phase with the other



interfering destructively when 3 waves are 1/3 out of phase, 4 waves are 1/4 out of phase, etc.

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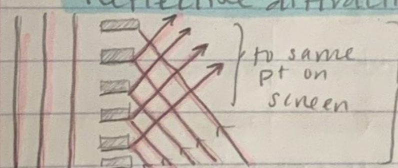
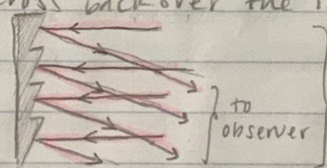
- Brightest fringes = principal maxima → correspond to constructive interference diffracted by all slits
- Fainter bright fringes = secondary maxima → correspond to incomplete cancellations
- For N slits, complete destructive interference occurs when waves differ in phase by $\frac{1}{N}$
  - Then, dark fringes occur when $d\sin\theta_k = \pm\left(\frac{k}{N}\right)\lambda$, where k is any non-whole number multiple of N
- Bright fringe location is the same regardless of number of slits, but there are N-1 dark fringes between bright fringes and N-2 secondary maxima between each pair of principal maxima
  - Brightness of fringes increases as number of slits increases because fringe brightness is determined by intensity (power/area) that strikes the screen, which increases with increased number of slits
- Diffraction gratings = barriers with large numbers of s
  - Transmissive diffraction gratings do not cross back over the incident ray
  - Reflective diffraction gratings do cross back over the incident ray



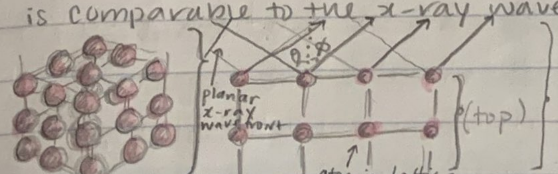
Transmissive diffraction grating



Reflective diffraction grating i.e. Compact disk

34.3 - X-Ray Diffraction

- X-Rays - EM waves with wavelengths from 0.01nm - 10nm (100x less than visible light)
  - small enough to pass through soft tissue (skin) and reflect solid materials (bone)
  - x-ray diffraction can be used to study atomic arrangements in solids
- Crystal lattice acts as a diffraction grating for x-rays because lattice spacing is comparable to the x-ray wavelength



when $\theta = \phi$, the rays have the same path length and therefore interfere constructively (or when $\delta_s = m\lambda$

- δ_s (for crystal lattice x-ray diffraction) $= 2d\cos\theta$, where d = distance between adjacent planes) and θ = angle of incidence

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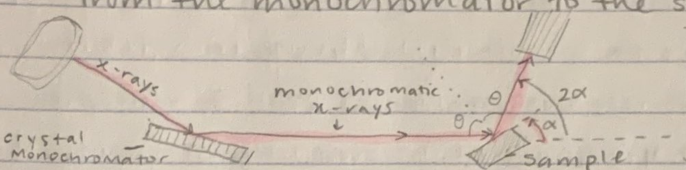
- Therefore, the condition for constructive interference is $2a\cos\theta = m\lambda$, which is called the Bragg Condition
- Crystal Monochromator = crystal of known lattice spacing
  - only one of the wavelengths hits the crystal in a way that satisfies the Bragg condition, so that monochromatic beam of x-rays is diffracted from the monochromator to the sample



θ = incident angle
α = Bragg angle (angle between x-rays and sample surface)

- X-Ray crystallography - used to determine the atomic structure of molecules

34.4 - Matter Waves

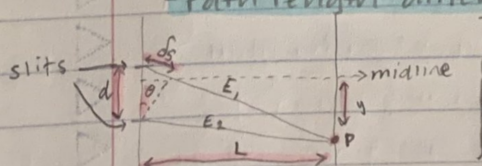
- Electrons exhibit both particle and wave behavior in the same slit experiment
- de Broglie wavelength $\lambda = \frac{h}{p}$ and represents the wavelength of a particle, where h = Planck's constant

34.5 - Photons

- wave-particle duality is also exhibited for light waves (with photons)
  - ↳ For a photon of frequency f light energy $E = hf$
  - Photons represent the quantum of electromagnetic energy

34.6 - Multiple Slit Interference

- Since the point sources from 2 slits are in phase, the electric fields from the two waves that reach the screen differ in phase only due to the difference in distance each wave travels from slit to screen
  - Path length difference δs



$E_1 = E_0 \sin\omega t$ & $E_2 = E_0 \sin(\omega t + \phi)$ (where E_0 = amplitude of electric field and ϕ = phase constant

↳ $\phi = \frac{2\pi}{\lambda} d\sin\theta$

- Bright fringes then occur at $\phi_m = \pm m(2\pi)$ and at angles $\sin\theta_m = \pm \frac{m\lambda}{d}$
- Dark fringes then occur at $\phi_n = \pm(2n-1)\pi$ and at angles $\sin\theta_n = \pm \frac{(n-\frac{1}{2})}{d}\lambda$
- The amplitude E_{12} = sum of the 2 electric fields $= 2(E_0 \cos\frac{1}{2}\phi)$
- Intensity S of EM wave $\Rightarrow S = \frac{4E_0^2}{\mu_0 c} \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right) \sin^2\omega t$
  - $S_{av} = 4S_{0av}\cos^2\left(\frac{\pi d\sin\theta}{\lambda}\right)$

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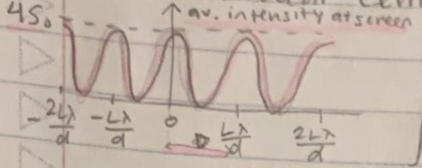
- The intensity of the EM waves varies periodically with the phase difference ϕ, and near central max (θ is small), intensity varies periodically with y

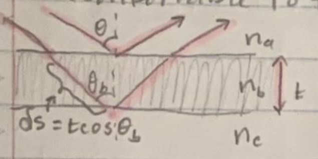


The distance D between adjacent maxima of this pattern is given by $D = \frac{L}{d}\lambda$

- The principal maxima occur at the same angles regardless of the number of slits, however the intensity at each maxima increases with the number of slits (and interference fringes become narrower)
- Two wavelengths can be distinguished from each other if the principal maxima of one falls in the first dark region of the other

34.7 - Thin Film Interference

- Thin film interference occurs in transparent materials whose thickness is comparable to the thickness of visible light (i.e. soap bubble)



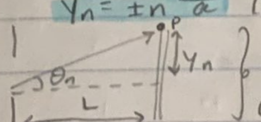
n_b = film index of refraction
t = film thickness
$\lambda_b = \frac{\lambda}{n_b}$ = wavelength inside film

wave inverted at n_a-n_b interface if $n_b > n_a$, and inverted at n_b-n_c interface if $n_c > n_b$

- The total phase difference between the two reflected waves is
  $\phi = \frac{4\pi n_b t \cos\theta_b}{\lambda} + \phi_{r2} - \phi_{r1} \Rightarrow \phi_{r1}$ & ϕ_{r2} must be either π or 0
  - ↳ $\phi = 0$ if reflected off surface with lower refractive index
  - $\phi = \pi$ if reflected off surface with greater refractive index

34.8 - Diffraction at Single Slit Barrier

- First order dark diffraction fringe is given by $\sin\theta_1 = \frac{\lambda}{a} \Rightarrow$ n-order: $\sin\theta_n = \pm n\frac{\lambda}{a}$
  - For fringes located at small angles θ_n from original wave propagation,
  $y_n = \pm n\frac{\lambda L}{a}$ (dark diffraction fringes)



w (distance between 2 minima) increases with decreasing a (slit width) $\Rightarrow$ narrower slit = more wave spreading after passing through

34.9 - Circular Apertures and Limits of Resolution

- Circular Aperture = circular diffraction pattern
  - Central bright fringe = Airy disk
- Circular Aperture of diameter $d \Rightarrow$ 1st dark fringe occurs at angle θ_1 given by:
  $\sin\theta_1 = 1.22\frac{\lambda}{d}$

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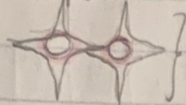
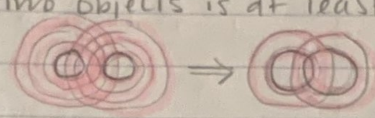
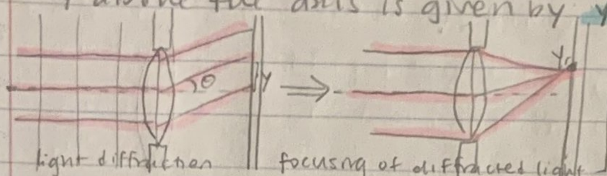
- **Rayleigh's Criterion** - Two objects are only resolved (distinguishable) if angular seperation between the two objects is at least $\sin\theta_r = 1.22\frac{\lambda}{d}$


} large aperture (stars resolved)


} small aperture (airy disks overlap ⇒ unresolved)

- Lens's focus diffracted light (because lens is an aperture), so distance Y above the axis is given by $y = f\tan\theta$ (where f = focal length of lens)

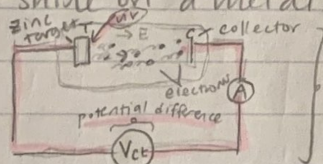


The angular size of Airy disk depends on both diffraction & focusing.
Radius of Airy Disk $y_r = 1.22\frac{\lambda f}{d}$

- The diffraction determined minimum size of the features in an image is called the diffraction limit

34.10 - Photon Energy and Momentum

- **Photoelectric Effect** - when photons of light of sufficiently high frequency shine on a metal, electrons are ejected from the metal



If V_{CT} is made slightly positive, a small current is detected but the electric field between T & C slows down the ejected electrons & as V_{CT} increases, there is a certain value of V_{CT} where electron flow completely stops, called the stopping potential difference.

- The change in electron KE given by: $\Delta K = -eV_{CT}$
- $K_{max} = K_i = eV_{stop}$ } max KE at which electrons leave the target
- $E_{photon} = hf = K_{max} + E_0$ } The photons energy frees the electron from the material & adds to its KE, so the minimum energy required to free the electron $= E_0 =$ work function of target metal & K_{max} = max KE of electron as it is ejected
- $V_{stop} = \frac{h}{e}f - \frac{E_0}{e}$ } where e = charge of electron
- $p_{photon} = \frac{E_{photon}}{c_0} = \frac{hf_{photon}}{c_0} = \frac{h}{\lambda_{photon}}$ } momentum of photon